Internal Reflections in Dielectric Prisms

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Summary—**A pair of closely spaced dielectric prisms used as an adjustable bidirectional coupler has been discussed by a number of investigators. In its simplest form this device permits an adjustable arbitrary distribution of power output in two directions at right angles to one another. Reflections from the external air-dielectric interfaces change the power distribution from the theoretically computed values and result in power output in a third direction. This paper describes a theoretical calculation of the power transmission in all four directions taking into account reflections at all interfaces. Experimental data recorded at 35 Gc exhibit very good agreement with predicted results.**

Introduction

THE DEVELOPMENT of sources of millimeter wavelength energy has increased interest in the applications of millimeter waves and has increased the frequency at which effective work can be done [1], [2].

High attenuation in dominant mode rectangular waveguide has made necessary the development of alternative transmission systems. One of these alternatives uses oversize waveguide in the TE_{01} circular mode [3], [4] and in the TE_{10} rectangular mode [5]. A second alternative makes use of a free-space transmission system with [6] or without beam-guiding lenses [7]. The use of quasi-optical (oversize) waveguide or free-space transmission systems requires the development of components and devices to perform all of the functions necessary in a transmission system [4], [5], [7], [8]. These devices include duplexers, directional couplers, corners, wavemeters, attenuators, and impedance meters.

A pair of dielectric prisms can be used to perform at least a part of all of the functions listed above in free space [7], [8], [9] and in oversize waveguide [15]. To review the functioning of the prisms, consider a pair of dielectric prisms as shown in Fig. 1. The dimension of the prisms normal to the paper is the same as the dimensions of the faces in the plan view. If reflections at all but the diagonal interfaces are neglected, a wave transmitted from a source located at position ① will be incident upon the dielectric air interface of the gap at an angle of 45°. The dielectric constant is chosen so that this angle is greater than the critical angle. If the prisms are widely separated, total reflection occurs at the gap, and all energy is transmitted in the direction of position ②. If the prisms are brought into contact so that the gap disappears, all energy is transmitted in the direction of position ③. By adjustment of the gap spacing, any desired division of power between the directions of positions ② and ③ can be obtained. Expressions for the transmission coefficient T and reflection coefficient R for electric field were developed by Schaefer and Gross [10] and can be expressed in the following form.

$$T = \frac{j2C}{(C^2 - 1)\sinh \alpha d + j2C \cosh \alpha d}$$

$$R = \frac{-(1 + C^2)\sinh \alpha d}{(C^2 - 1)\sinh \alpha d + j2C \cosh \alpha d}.$$

In the above expressions $C = \sqrt{(K \sin^2 \theta - 1)/K \cos^2 \theta}$. For waves polarized in the plane of incidence, C is replaced by KC.

K—Relative dielectric constant
θ—Angle of incidence
d—Gap spacing-meters
α—$\omega\sqrt{\mu_0\epsilon_0}\sqrt{K \sin^2 \theta - 1}$
ω—Angular frequency
μ_0, ϵ_0—Permeability and dielectric constant, respectively, for free space.

Several investigators have utilized the prisms as an adjustable bidirectional coupler or attenuator in free space [7], [11] and in quasi-optical guide [5]. In these instances quarter wave plates or slots have been used as

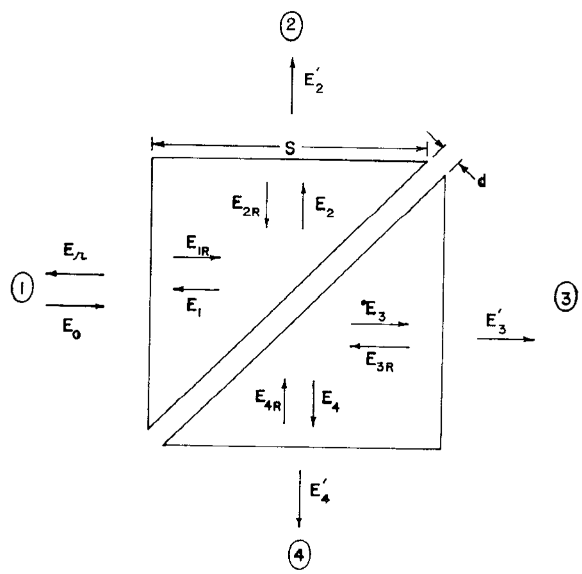

Fig. 1—Plan view of prisms indicating direction of wave propagation.

Manuscript received April 27, 1964; revised June 26, 1964. The investigation reported here has been made possible through the support of the U. S. Air Force Office of Scientific Research, Office of Aerospace Research, under Grant AF-AFOSR-3-63.

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matching devices on the outer faces of the prisms. For the specific case of $\theta = 45°$ the expression for C simplifies to $C = \sqrt{(K-2)/K}$.

Internal Reflections

At shorter wavelengths, physical dimensions make the fabrication of matching devices quite difficult. The effect of these reflections is to return some energy to the source, to alter somewhat the energy transmitted in the directions of positions ② and ③ and to produce some energy output in the direction of position ④.

In order to analyze these reflections, use is made of Fig. 1. In this figure E_r, E_0, E_1, E_{1R}, E_2, E_{2R}, E_2', E_3, E_{3R}, E_3', E_4, E_{4R}, and E_4' represent the complex amplitudes of the electric field in the waves traveling in the directions indicated in Fig. 1. Other symbols are defined as follows.

s—side dimension meters

γ—complex propagation constant of dielectric.

Γ—dielectric to air reflection coefficient for plane wave normally incident.

Γ'—air to dielectric reflection coefficient for plane wave normally incident.

Ψ—dielectric to air transmission coefficient for plane wave normally incident.

Ψ'—air to dielectric reflection coefficient for plane wave normally incident.

The signal flow graph representing the relations between the variables of interest is given in Fig. 2. The complex field amplitudes represented by the capital E's are the variables represented by the nodes in the flow graph, and the path transmission coefficients are indicated in the flow graph.

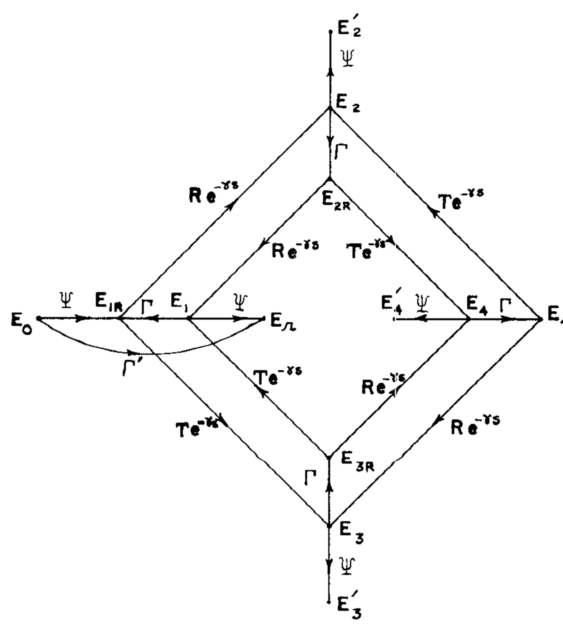

Fig. 2—Signal flow graph for determining transmission coefficients.

$$\frac{E_2'}{E_0} = \Psi\Psi' \, Re^{-\gamma s} \frac{1-(R^2-T^2)\Gamma^2 e^{-2\gamma s}}{\Delta}$$

$$\frac{E_3'}{E_0} = \Psi\Psi' Te^{-\gamma s} \frac{1+(R^2-T^2)\Gamma^2 e^{-2\gamma s}}{\Delta}$$

$$\frac{E_4'}{E_0} = \frac{2\Psi\Psi' RT\Gamma e^{-2\gamma s}}{\Delta}$$

$$\Delta = 1 - 2(R^2+T^2)\Gamma^2 e^{-2\gamma s} + (R^2-T^2)^2\Gamma^4 e^{-4\gamma s}.$$

Computed Results

The IBM 1620 computer was used to compute the transmission coefficients above as a function of gap spacing and side dimensions measured in wavelengths. Computations were carried out for polyethylene ($K = 2.26$ and loss tangent 0.0007) as well as for several other values of K.

Fig. 3 includes curves showing the three transmission coefficients as a function of gap spacing for waves polarized normal to the plane of incidence and Fig. 4 provides similar data for waves polarized in the plane of incidence.

Figs. 5 and 6 show the variation of the transmission coefficients for waves polarized normal to the plane of incidence as a function of prism side length for two different dielectric constants, assuming no dielectric loss. In this case the gap is set at 0.5 wavelength.

In these figures

$$T_{02} = 20 \log_{10} \frac{|E_2'|}{|E_0|}$$

$$T_{03} = 20 \log_{10} \frac{|E_3'|}{|E_0|}$$

$$T_{04} = 20 \log_{10} \frac{|E_4'|}{|E_0|}.$$

Experimental Results

Experimental work was done at several frequencies near 35 Gc using a Raytheon QK 291-reflex klystron with appropriate square-wave modulated power supply. Well-matched horns were used as transmitting and receiving antennas with suitable matched crystal detectors. The prisms were halves of a nine-inch cube of polyethylene.

Relative power measurements were made using a Polytechnic Research and Development Company calibrated attenuator type 192 F1. In the measurement of transmission loss power was maintained at a constant level by use of a directional coupler monitor and an adjustable attenuator. In obtaining these data, the precision attenuator was set at some high value to yield a convenient reading on the meter connected to the receiver output with the prism spacing set at zero. As the gap was increased by closely measured increments, the attenuator was readjusted to give the same power out-

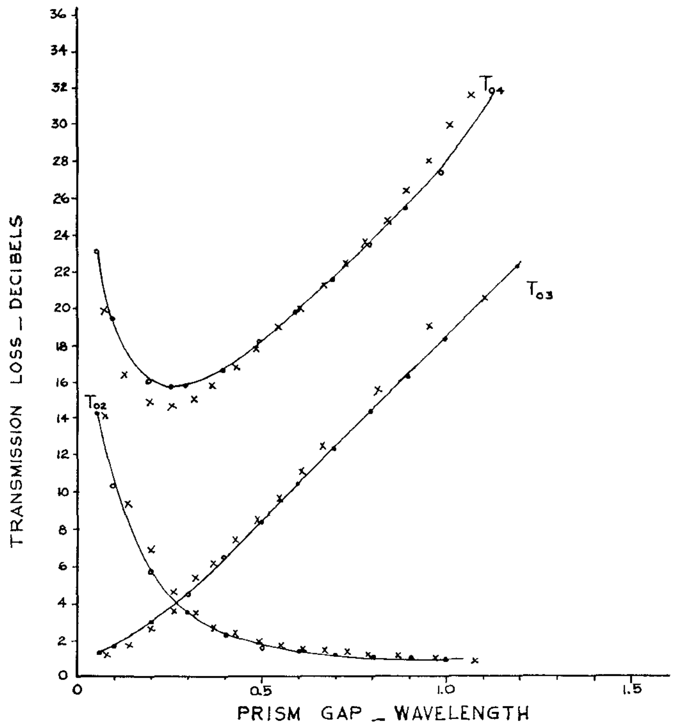

Fig. 3—Transmission coefficients of the prism device including the effect of internal reflections for waves polarized normal to the plane of incidence at a frequency of 35.3 Gc. Circles represent theoretically computed points, and crosses represent experimental points.

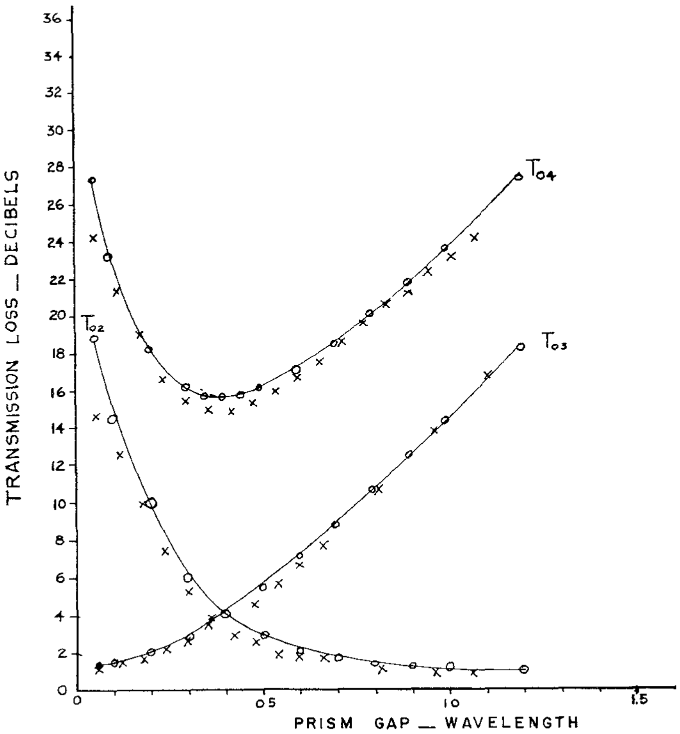

Fig. 4—Transmission coefficients of the prism device including the effect of internal reflections for waves polarized in the plane of incidence at a frequency of 35.84 Gc. Circles represent theoretically computed points, and crosses represent experimental points.

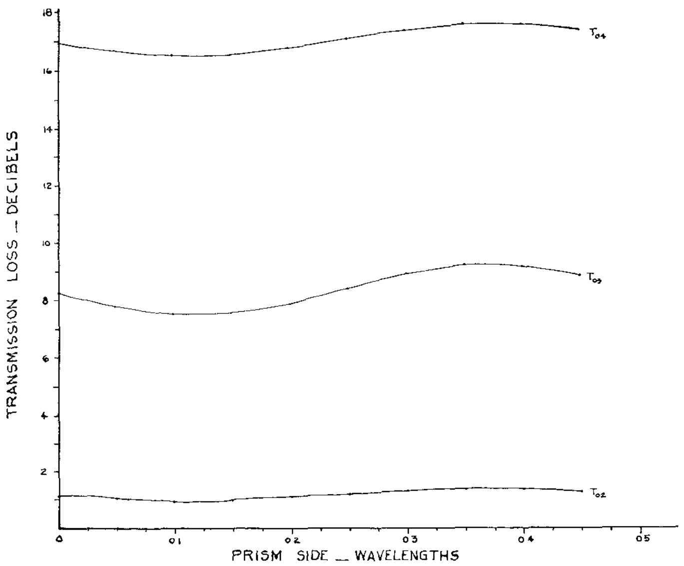

Fig. 5—Theoretically computed transmission coefficients of the prism device plotted as a function of prism dimensions for a gap width of 0.5 wavelength, for waves polarized normal to the plane of incidence, and for a relative dielectric constant of 2.26.

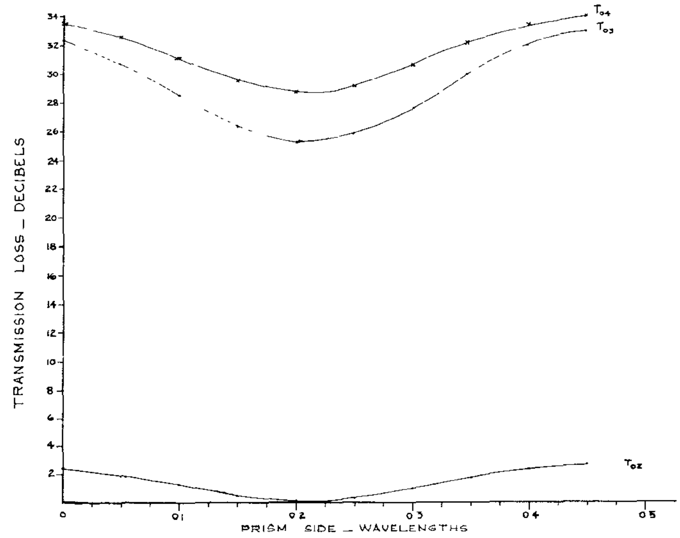

Fig. 6—Theoretically computed transmission coefficients of the prism device plotted as a function of prism dimensions for a gap width of 0.5 wavelength, for waves polarized normal to the plane of incidence, and for a relative dielectric constant of 5.00.

put reading. This serves to eliminate crystal law and amplifier nonlinearity as possible sources of error. As a zero check, loss was measured with the prisms removed. Experimental data are plotted in Figs. 3 and 4. Due to uncertainty of the zero position because of bowing of the prism faces, the experimental curves have been shifted (0.06–0.08 wavelength) so that the positions of the minima in T_{04} in the experimental and computed results coincide.

Discussion of Results

The accuracy of the measurements was limited to the accuracy of the 192 F1 attenuator. It was not possible to work with the horns at a great distance from the prisms so some error is attributable to the spherical wave front. As the prisms were of finite size, some evidence of diffraction around the prisms was observed. Due to the tendency of the diagonal prism faces to assume a bowed position, it was impossible to obtain a gap of zero width and the gap was not of perfectly uniform width along its entire length. As the curves indicate, there is good general agreement between the experimental results and the behavior predicted theoretically. It should be noted that transmission to a receiver at location ④ occurs only in the presence of internal reflections.

Conclusion

The prism device is useable as an adjustable bidirectional coupler and as an adjustable attenuator. With coupling set at three decibels, it can be used as a hybrid junction. The present paper permits the theoretical calculation of coupling in all directions in the presence of internal reflections. The adaptability of the prism device to a wide variety of functions makes it very useful at the shorter millimeter wavelengths where the elimination of internal reflections is difficult. The calculations presented here should prove to be applicable in this case.

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Nonreciprocal Coupling with Single-Crystal Ferrites

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Summary—**A calculation of nonreciprocal coupling in microwave circuits with small ferrite samples tuned to ferromagnetic resonance is presented. It is shown that this coupling may be applied to the construction of simple resonant isolators, gyrators and circulators. Experimental results for the coupling in rectangular and ridge guides, applying YIG spheres, are presented. The construction of a simple *X*-band waveguide junction, acting as a 4-port resonant circulator, is described. Such a filter circulator, which may act as a switch or a frequency selective power divider, can be made tunable over the waveguide frequency range, with a bandwidth in the order of 10 Mc, and with values of insertion loss and isolation, which are comparable to those of conventional circulators.**

Introduction

IN RECENT YEARS, considerable attention has been given to polished ferrimagnetic single crystals for the construction of narrow band filters and microwave power limiters. This is due to their extremely low losses when used as resonating elements in filters, and the fact that coupling to microwave circuits may be achieved in a very simple way. Also, magnetic tuning may be accomplished over a wide band of frequencies.

Manuscript received April 27, 1964; revised July 13, 1964.
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Several constructions of band-pass and band-rejection filters have been reported,[1,2] but little attention has been paid to nonreciprocal devices, containing high Q single-crystal ferrite.[3] The band-pass filters reported by Carter[1] and others are nonreciprocal in the sense that there is a $+90°$ phase shift in one direction of propagation, and a $-90°$ phase shift in the other direction, but this nonreciprocity is not primarily wanted.

The first part of this paper consists of an analysis of the nonreciprocal coupling caused by a small single crystal sample placed in an elliptically polarized microwave magnetic field, when the sample is tuned to ferromag-

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